Appraisal Institute™

Professionals Providing Real Estate Solutions

Review of Necessary Mathematics and Analytical Skills

REVIEW OF NECESSARY
MATHEMATICS AND ANALYTICAL SKILLS

I. ORDER OF OPERATIONS

Background—A universal agreement exists regarding the order in which addition, subtraction, multiplication, and division should be performed.

1) Powers and roots performed first.

2) Multiplication and division are performed next from left to right in the order that they appear.

3) Additions and subtractions are performed last from left to right in the order that they appear.

Example 1:

$$3 + 4 \times 5 = $$
$$3 + 20 = 23$$

Example 2:

$$7 \times 3^2 = $$
$$7 \times 9 = 63$$

Note. If grouping symbols such as parentheses "( )," brackets "[ ]," and braces "{}," are present, the operations are simplified by first starting with the innermost grouping symbols and then working outward.

Example 3:

$$\{[12 - 2 \times (7 - 2 \times 2)] \div 3\}^2 =$$
$$\{[12 - 2 \times (7 - 4)] \div 3\}^2 =$$
$$\{[12 - 2 \times 3] \div 3\}^2 =$$
$$\{[12 - 6] \div 3\}^2 =$$
$$\{6 \div 3\}^2 =$$
$$2^2 = 4$$

II. SUBTRACTING/ADDING NEGATIVE NUMBERS

Background—Every negative number has its positive counterpart, which is sometimes called its additive inverse. The additive inverse of a number is that number which when added to it produces 0. Thus, the additive inverse of −5 is +5 because (−5) + (+5) = 0. Subtracting a negative number is the same as adding its positive counterpart. Adding a negative number is the same as subtracting its positive counterpart.

Example 1:

$$17 - (-3) =$$
$$17 + (3) = 20$$

Example 2:

$$12 + (-8) - (-10) =$$
$$12 + (-8) + (10) =$$
$$12 + (-8) + (10) = 14$$

III. MULTIPLICATION/DIVISION WITH NEGATIVE NUMBERS

Background—When numbers of opposite signs are multiplied or divided, the result is negative. When numbers of the same sign are multiplied or divided, the result is always positive. When dividing or multiplying, the two negative signs cancel out.

Example 1:

$$6 \times (-7) \div 3 =$$
$$(-42) \div 3 = (-14)$$

Example 2:

$$(-32) \div (-4) = 8$$

IV. ADDITION/SUBTRACTION OF FRACTIONS

Background—Simplifying fractions by addition or subtraction requires the use of the lowest common denominator. The denominator on both fractions must be the same before performing an operation. Just as when adding dollars and yen, the yen must be converted to dollars before addition.

Example 1:

$$\$120 + ¥13{,}000 =$$

$$\$120 + ¥13{,}000\left(\frac{dollars}{130\ \ Yen}\right) =$$

$$\$120 + \$100 = \$220$$

Example 2:

$$\frac{3}{4}+\frac{2}{3}=$$

$$\left(\frac{3}{3}\right)\frac{3}{4}+\left(\frac{4}{4}\right)\frac{2}{3}=$$

$$\frac{9}{12}+\frac{8}{12}=\frac{17}{12}$$

V. MULTIPLICATION/DIVISION OF FRACTIONS

Background—Multiplication with fractions is very straightforward, just multiply numerator by numerator and denominator by denominator. When dividing with a fraction, the number being divided (dividend) is multiplied by the reciprocal of the divisor. Frequently this has been stated "invert and multiply."

Example: $3 \div \frac{3}{4} = 3 \times \frac{4}{3} = \frac{12}{3} = 4$

VI. COMPOUND FRACTIONS

Background—Frequently a mathematical expression appears as a fraction with one or more fractions in the numerator and/or the denominator. To simplify the expression multiply the top and bottom of the fraction by the reciprocal of the denominator.

Example:

$$\frac{\frac{2}{5}}{4} =$$

$$\frac{\frac{1}{4} \times \frac{2}{5}}{\frac{1}{4} \times 4} = \frac{\frac{2}{20}}{1} =$$

$$\frac{2}{20} = \frac{1}{10}$$

Note. When multiplying or dividing the numerator and denominator of the fraction by the same number the value of the fraction does not change. In essence, the fraction is being multiplied/divided by 1.

VII. EXPONENTS

Background—Exponents were invented to make it easier to write certain expressions involving repetitive multiplication: $K \times K \times K \times K \times K \times K \times K \times K \times K = K^9$. Note that the exponent (9) specifies the number of times the base (K) is used as a factor rather than the number of times multiplication is performed.

Example: $6^4 = 6 \times 6 \times 6 \times 6 = 1{,}296$

VIII. FRACTIONAL EXPONENTS

The definition of a fractional exponent is as follows:

$$X^{M/N} = \sqrt[N]{X^M}$$

This equality converts an expression with a radical sign into an exponent so that the y^x key found on most financial calculators can be used.

Example 1: $12^{4/5} = \sqrt[5]{12^4} = 7.3009$

Example 2: $\sqrt[5]{10} = 10^{1/5} = 10^{0.2} = 1.5849$

IX. SUBSCRIPTS/SUPERSCRIPTS

Background—Concepts or variables that are used in several equations generally use subscripts or superscripts to differentiate the values.

Example: Capitalization rates are expressed as a capital "R." Since there are a number of different capitalization rates used by appraisers, a subscript is used to specify which capitalization rate is intended. An equity capitalization rate, therefore, is written as R_E.

X. PERCENTAGE CHANGE

Background—Calculating percentage change or delta "Δ" is required in several of the capitalization techniques. The formula for "Δ" is:

$$\Delta = \frac{\text{final value} - \text{starting value}}{\text{starting value}}$$

Example: What percentage of change occurs if a property purchased for $90,000 sells for $72,000?

Answer:

$$\Delta = \frac{\$72{,}000 - \$90{,}000}{\$90{,}000} =$$

$$\Delta = \frac{-\$18{,}000}{\$90{,}000} = -0.20 = -20\%$$

Example 2: What percentage of change occurs if a property purchased for $75,000 sells for $165,000?

Answer:

$$\Delta = \frac{\$165{,}000 - \$75{,}000}{\$75{,}000} =$$

$$\Delta = \frac{\$90{,}000}{\$75{,}000} = 120\%$$

XI. CANCELLATION OF UNITS

Background—Many appraisal applications involve the multiplication and/or division of numbers with "units" associated with them, e.g., $/sf, sf, ft, yr., etc. The proper handling of these units is necessary to correctly describe the mathematical result. According to the identity principle, any number/variable divided by itself is equal to 1 and can thus be removed from the equation.

$$\frac{5}{5} = \frac{X}{X} = \frac{7xy^2}{7xy^2} = 1$$

Example: What value would be indicated for a 12,000 sf building if it is worth $55/sf?

Answer:

$$12{,}000 sf \times \frac{\$55}{sf} = \$660{,}000$$

$$sf \times \frac{\$}{sf} = \$$$

XII. SOLVING EQUATIONS

Background—In many instances, an equation or formula exists in a form that is not convenient for the problem at hand, e.g., with value as the goal and the available equation is: I = R × V. Using equation solving techniques, the formula can be rewritten to solve for value with V = I ÷ R as the result.

The rules of equation solving are quite simple and are as follows:

1) Adding or subtracting the same number/variable to both sides of the equation will not change the solution.

2) Multiplying or dividing both sides of the equation by the same number/variable will not change the solution.

3) Raising both sides of the equation by the same power or taking the same root will not change the solution.

Example:

$$Income = Capitalization\ Rate \times Value$$

$$I = R \times V \qquad \text{Divide both sides by R}$$

$$\frac{I}{R} = \frac{R \times V}{R}$$

$$\frac{I}{R} = V$$

A caution to be noted! Multiplying both sides of an equation by an expression containing a variable or the unknown *could* result in an equation with additional roots that were not in the original equation. This does not change the answer of either equation, nor does it simplify the answer. For example x = 5 has one root, 5. Multiplying both sides by "x" results in $x^2 = 5x$ which has two roots, 0 and 5. Also, dividing both sides of an equation by an expression containing a variable or the unknown could result in an equation losing roots that were in the original equation. For example, $x^2=5x$ has two roots, 5 and 0. Dividing both sides by "x" results in x=5, which has one root 5.

XIII. PROBLEM SOLVING

Background—Formal problem solving techniques vary from person to person, but usually fall into a sequence of steps that can be categorized as follows:

1) Identify the question to be answered. If the required solution can be represented by a symbol, write it as such. Since many problems require the use of a formula, the identification of information in symbol form helps one recognize potential formula(s) that might be used to solve the problem.

2) Carefully glean all of the given data from the problem statement and assign symbols, if possible. The data may be represented as a number, a word, or a phrase ($10,000, six, value will double during the projection period).

3) Based on the identification and assignment of symbols in steps 1 and 2, attempt to list (mentally or on paper) all of the methods or techniques (frequently a formula) that you are aware of that can be used to find the answer.

4) Compare the list in step 3 with the data from steps 1 and 2. Based on this comparison, one of the following situations will emerge:

    a) The solution is fairly obvious and all of the necessary information has already been identified.

    b) The solution is fairly obvious but some additional data must be created from the given information.

    c) The solution is not obvious and the items in the list in step 3 must be considered one-by-one until a correct one is found.

    d) "a," "b," and "c" fail to solve the problem. Steps 2 and 3 may have been improperly handled and must be revisited with "a," "b," and "c" retried. In the worst case scenario (not in an Appraisal Institute course), the problem may not be solvable.

Example: What is the present value of $1,500 per year for 12 years discounted at 15%?

Step 1: Solve for present value or PV.

Step 2: Identify all variables:

Years	=	12
Discount rate	=	15%
Annuity	=	$1,500

Step 3: Possible equations: $PV = CF\left[\frac{1-\left(\frac{1}{(1+i)^n}\right)}{i}\right]$

Step 4: With only one possible equation and all the variables accounted for this problem becomes straightforward.

$$\text{PV} = \$1{,}500\left[\frac{1-\left(\frac{1}{(1+0.15)^{12}}\right)}{0.15}\right]$$

$$= \$1{,}500\left[\frac{1-\left(\frac{1}{5.35025}\right)}{0.15}\right]$$

$$= \$1{,}500\left[\frac{0.813093}{0.15}\right]$$

$$= \$1{,}500\,[5.4206]$$

$$= \$8{,}130.93$$